2024-01-24

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Elsevier

https://dspace.nm-aist.ac.tz/handle/20.500.12479/2528

Chaos, Solitons & Fractals: X 12 (2024) 100105

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals: X

journal homepage: www.elsevier.com/locate/csfx

Parameters estimation, global sensitivity analysis and model fitting for the dynamics of *Plutella xylostella* infestations in a cabbage biomass

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ARTICLE INFO

Keywords:
Plutella xylostella
Brassica olerecea and predators

ABSTRACT

Plutella xylostella, commonly called Diamondback moth (DBM), a highly destructive and rapidly spreading agricultural pest originally from Europe. This pest poses a significant threat to global food security, with estimates suggesting that periodic outbreaks of Diamondback moth lead to annual crop losses of up to $US 4 – 5 billion worldwide. Given the potential for such substantial losses, it is crucial to employ various methods and techniques to understand the factors affecting the interaction between Diamondback moths and cabbage plants, which, in turn, impact cabbage biomass. In this paper, we propose a deterministic ecological model to capture the dynamics of *Plutella xylostella* infestations in cabbage biomass. The model is designed based on the life cycle stages of the pest, aiming at targeting the specific stage effectively. The synthetic data is generated using Least Square Algorithm through addition of Gaussian noise into numerically obtained values from existing literature to simulate real-world data. Global sensitivity analysis was done through Latin Hypercube sampling, highlights the significance of parameters such as ψ, α_E and δ positively influence the growth of the diamondback moth in a cabbage biomass. In light of these findings, the study proposes that control strategies should be specifically directed towards these sensitive parameters. By doing so, we mitigate the pest population and enhance cabbage production.

1. Introduction

Cabbage is the nutritious vegetable which offers various health benefits such as vitamins and minerals to the human body [1]. It has been associated with lower incidences of chronic diseases such as cancer and heart diseases [2,3]. Additionally, as an agricultural produce, cabbage plays a vital role in providing food security and income for farmers, contributing to a nation's foreign currency earnings [4]. Despite of all its benefits, cabbages are affected by various pest infestations such as cutworms, diamondback moths, and plant diseases, which lead to significant yield losses [5].

Plutella xylostella, commonly called Diamondback Moth (DBM), stands as a highly destructive agricultural pest, presenting substantial challenges to cruciferous crops, particularly cabbages [6]. The female moths deposit their eggs on cabbage leaves, and after some days eggs hatch to larva which cause significant harm by burrowing into the leaf tissue, as depicted in Fig. 1. This feeding behavior leads to distinct diamond-shaped holes in the leaves, which, in turn, results in decreased plant health, stunted growth, and lower crop yields [7].

The management of *Plutella xylostella* in cabbages brings significant financial burdens in various regions. For example, China allocates approximately USD 0.77 billion annually for control effort. In Africa, the weekly cost for managing *Plutella xylostella* is estimated to be about USD 46 097 772 [9]. These financial burdens, coupled with expenses related to insecticide use and genetically modified cabbage varieties is a big challenge to countries like Tanzania [10,11].

Integrated pest management (IPM) represents a strategy employed by farmers to combat pest infestations by utilizing various pest control tactics, as documented in numerous studies [12–17]. These approaches include biological control, encompassing the utilization of predators and parasitoids. Predators, such as lady beetles and lacewings, are self-sustaining species that, over their lifespan, consume a substantial quantity of prey. Conversely, parasitoids such as *Cotesia vetalis*, develop either within or on an insect host, ultimately resulting in the

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https://doi.org/10.1016/j.csfx.2024.100105
Received 12 November 2023; Received in revised form 3 January 2024; Accepted 24 January 2024

Fig. 1. The effects of larva infestations on a cabbage biomass [8].

host's demise [18–20], implementing cropping practices like rotation and inter-cropping [21–23], and applying insecticides [24–26].

Mathematical modeling techniques provide valuable insights into the dynamics of population species and their interactions with the environment, for example, modeling help us in managing the pest like *Plutella xylostella*. The scholars such as Faithpraise et al. [27], Alexandridis et al. [28], Alarcón-Segura et al. [29] proposed deterministic models to study pest infestations in different crops including cabbages. On the other side of the coin, Ruttanaprommarin et al. [30] a proposed a stochastic mathematical models to analyze predator-prey models by using delay differential Equations Holling type-III. The study done by Umar et al. [31], employed a non-linear mathematical models to analyze predator-prey interactions by using an artificial neural network. In a different study, Daudi et al. [32] proposed a fractional order predator model to capture the dynamics of invasive pest in a maize crops. In similar vein, Sabir et al. [33] developed a Fractional order model to capture the predator prey interactions by using stochastic procedures with the application of artificial neural networks. On the other hand, the scholars such as Marchioro et al. [34], Hariprasad and Van Emden [35], employed Statistical approaches to predict DBM behavior in field conditions and larvae resistance to cypermethrin insecticides, whereas Do Carmo et al. [36], Mookiah et al. [37], have assessed the effectiveness of commercial pesticides against *Plutella xylostella*, considering their impact on beneficial insects. However, pesticides can hinder the life cycle of the pests but still have detrimental effects on beneficial insects, particularly predator ants.

The aforementioned studies and several other cited therein have certainly produced many useful results and improved the existing knowledge on plant–pest interaction such as cabbage-*Plutella xylostella*. However, their model formulation did not consider the life cycle stages of the *Plutella xylostella* and the estimation of the parameters using Least Squares Algorithm was not performed, specifically the parameters that drive the growth of *Plutella xylostella*.

To fill that gap, this study estimated the parameters by employing the non-Linear Least Square Method with addition of Gaussian noise to the literature values so as to reflect the real world data for the dynamics of the *Plutella xylostella* in a cabbage farms. Finally, we check the influences of parameters to the growth of *Plutella xylostella* by using Latin Hypercube sampling method.

This manuscript is organized as follows: Section 2 captures the model formulation and its properties, Section 3 details the parameters estimations, Section 4 presents numerical analysis, including graphs and global sensitivity analysis, Section 5 gives the discussions of the results and Section 6 provides concluding remarks.

2. Methodology of the study

2.1. Deterministic model formulation

The present model proposes a deterministic ordinary differential equations (ODEs) that describe three distinct populations: cabbage biomass, $C(t)$; the *Plutella xylostella* population comprising of Egg, $E(t)$; Larva, $L(t)$; and Adult stage, $A(t)$; and the predator population, $H(t)$. While *Plutella xylostella* population is age-structured pest, the typical *Plutella xylostella* life cycle includes four stages, we have simplified it to three by extending the transition rate of larval stage to adult moth stage (that extension makes pupae stage captured), making our model consisting of five compartments.

The *Plutella xylostella* life cycle begins with the deposition of eggs which are usually in clusters on cabbage biomass, predominantly on the undersides of leaves. The Eq. (1) delineates the dynamics of eggs within the cabbage biomass.

$$\frac{dE}{dt} = \psi q\left(1 - \frac{E}{K_E}\right)A - (\lambda_E + \alpha_E)E. \tag{1}$$

In this equation, q denotes the average deposition of eggs laid per female adult *Plutella xylostella* per day, where as K_E signifies the capacity for egg deposition, indicating the available space for laying eggs. The parameter ψ represents the proportion of adult female *Plutella xylostella*, while α_E represents the egg hatching rate and λ_E is the egg-natural mortality.

After the period of $2 - 9$ days the eggs turn to *Plutella xylostella* larvae. We present the infestations dynamics of the larval on cabbages by Eq. (2);

$$\frac{dL}{dt} = \alpha_E E + \eta\omega LC - (\lambda_L + \delta)L - \beta_h LH. \tag{2}$$

The parameter α_E represents the transition rate from the egg stage to larvae. We assume that λ_L represents the natural larval mortality rate, and δ signifies the average duration of the larval stage until becoming an adult moth, estimated at $14 - 16$ days. The term $\eta\omega LC$ describes the interaction between larvae and cabbage biomass, leading to the conversion of cabbage biomass into larvae biomass, where η represents the efficiency of this conversion.

Adult *Plutella xylostella* are responsible for laying eggs on cabbage biomass, either on the surface or underneath cabbage leaves. On average, a female DBM lays about 160 eggs during its lifespan, which hatch into larvae after six days and eventually develop into adults within $15 - 18$ days. The population dynamics of adult DBM is summarized by Eq. (3);

$$\frac{dA}{dt} = \delta L - \lambda_A A. \tag{3}$$

In this regard, δ accounts for the proportion of larvae successfully progressing to the adult stage, and λ_A represents the average lifespan of DBM, typically around 18 days but ranging from 15 to 18 days.

Moreover, under favorable conditions, *Plutella xylostella* larvae can feed on cruciferous vegetables like cabbages. Accounting all cabbage varieties, cabbages are planted at time $t = 0$ and taking $60 - 180$ days to mature. We represent the cabbage biomass per plot as C(t), and its dynamics described in Eq. (4);

$$\frac{dC}{dt} = r\left(1 - \frac{C}{K_C}\right)C - \omega LC. \tag{4}$$

In this equation, r signifies the growth rate of cabbage biomass, K_C represents the maximum biomass of cabbage, and ω is the rate at which *Plutella xylostella*-larvae attack the cabbage biomass such as leaves.

To mitigate the severe effects of larvae on cabbage biomass, we introduce predators such as *Cotesia vetalis*, birds, and lady beetles into the cabbage farm. The parameter β_h captures the predator's attack rate on larvae. The book written by study done by Capinera [38] and an article by Rafikov et al. [39], suggests that when food (larvae) is limited,

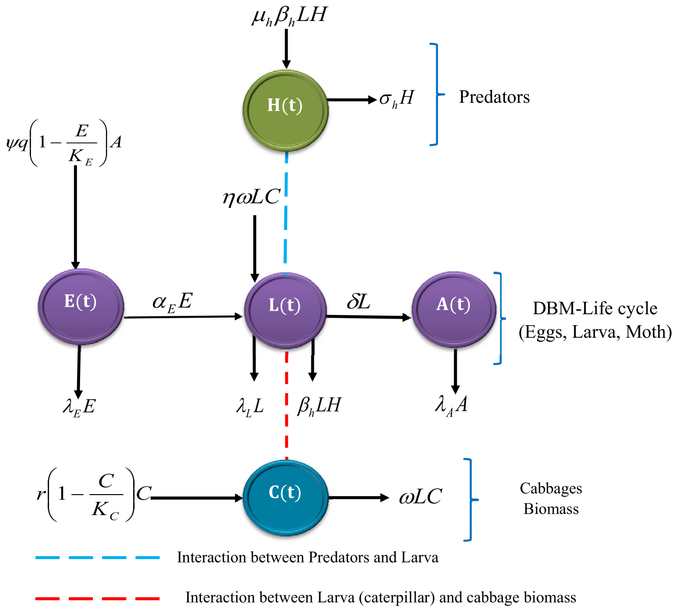

Fig. 2. The model diagram for the dynamics of *Plutella xylostella* and Predators in a cabbage biomass.

predators turn to prey, which is captured by the parameter μ_h, representing the conversion rate of predators into prey while σ_h is the life span of the predators. The population of predators is mathematically defined in Eq. (5);

$$\frac{dH}{dt} = \mu_h\beta_h LH - \sigma_h H. \tag{5}$$

The entire scenario is summarized following the map flow in Fig. 2 and by system of non-linear ODEs, Eq. (6), capturing the interactions between these populations and their dynamics on cabbage biomass;

$$\begin{cases} \dfrac{dC}{dt} = r\left(1-\dfrac{C}{K_C}\right)C - \omega LC, \\ \dfrac{dE}{dt} = \psi q\left(1-\dfrac{E}{K_E}\right)A - (\lambda_E+\alpha_E)E, \\ \dfrac{dL}{dt} = \alpha_E E + \eta\omega LC - (\lambda_L+\delta)L - \beta_h LH, \\ \dfrac{dA}{dt} = \delta L - \lambda_A A, \\ \dfrac{dH}{dt} = \mu_h\beta_h LH - \sigma_h H, \end{cases} \tag{6}$$

with initial values,

$$C(0)\geq 0, E(0)\geq 0, L(0)\geq 0, A(0)\geq 0, H(0)\geq 0.$$

2.2. Boundedness of the model solution

Theorem 1. *The ecological deterministic model shown by system (6) has a unique solution in $\mathbb{R}^5_+$ which lies in a region Ω with sub-regions Ω_c, Ω_x, and Ω_h, such that;*

$$\begin{aligned} \Omega_c &= \left\{C\in\mathbb{R}_+ \mid 0\leq C\leq K_C\right\}, \\ \Omega_x &= \left\{(E,L,A)\in\mathbb{R}^3_+ \mid \Theta\leq\frac{x}{h}(\psi q+1)\left(1-e^{-ht}\right)+\Theta(0)e^{-ht}\right\}, \\ \Omega_h &= \left\{H\in\mathbb{R}_+ \mid L=\frac{\sigma_h}{\mu_h\beta_h}\right\}. \end{aligned} \tag{7}$$

Proof. The model consists of three populations: the cabbage biomass, the diamondback moth stages and the population of predators. We initiated our proof by using the function $\Theta(t)$, as depicted in Eq. (8), to represent the population of diamondback moth at various stages.

$$\Theta(t) = E(t)+L(t)+A(t), \tag{8}$$

then from Eq. (8), we have,

$$\begin{aligned} \frac{d\Theta(t)}{dt} &= \frac{dE(t)}{dt}+\frac{dL(t)}{dt}+\frac{dA(t)}{dt}, \\ &\leq A(\psi q+1)-\left(\alpha_1 L+\alpha_2 A\right), \\ &\leq m(\psi q+1)-h\Theta, \text{ for } m=\max\left(E(0),M\right) \text{ and } h=\min\left(\alpha_1,\alpha_2\right). \end{aligned} \tag{9}$$

$$\frac{d\Theta(t)}{dt}+h\Theta\leq m(\psi q+1). \tag{10}$$

Solving Eq. (10) analytically we have the following:

$$\Theta(t)\leq\frac{m}{h}(\psi q+1)\left(1-e^{-ht}\right)+\Theta(0)e^{-ht}. \tag{11}$$

As $\lim_{t\to\infty}\Phi(t)$, then the solution of Eq. (11) becomes;

$$\Phi(t)\leq\frac{m}{h}(\psi q+1). \tag{12}$$

Similarly, to assess the boundedness of the predator population we solved the equation for predators, and we obtained the condition for its boundedness shown in Eq. (13):

$$L=\frac{\sigma_h}{\mu_h\beta_h}. \tag{13}$$

Therefore, utilizing Theorem 1 the model system Eq. (6) is mathematically and ecologically well posed in a region Ω. ☐

2.3. Equilibrium points of the model

In this subsection, we investigate the presence of equilibrium points for the system denoted by Eq. (6). Specifically, we identify a total of five equilibrium points that are positive in nature. i.e. $\mathcal{U}_1$, $\mathcal{U}_2$, $\mathcal{U}_3$, $\mathcal{U}_4$, and $\mathcal{U}_5$:

(a) Equilibrium point $\mathcal{U}_1\,(0,0,0,0,0)$. This is trivial equilibrium point and always exist.
(b) Pest free equilibrium point $\mathcal{U}_2\left(K_C,0,0,0,0\right)$. This always exists.
(c) Equilibrium point $\mathcal{U}_3\left(0,E_3,L_3,A_3,H_3\right)$, such that

$$\begin{cases} E_3 = \frac{K_E\psi q\sigma_h\delta}{K_E\mu_h\beta_h f_1\lambda_A+\psi q\sigma_h\delta}, \\ L_3 = \frac{\sigma_h}{\beta_h\mu_h}, \\ A_3 = \frac{\sigma_h\delta}{\beta_h\mu_h\lambda_A}, \\ H_3 = \frac{1}{\beta_h}\left(f_2-\alpha_E\right), \end{cases}$$

where

$$\begin{aligned} f_1 &= \alpha_E+\lambda_E, \\ f_2 &= \delta+\lambda_L, \end{aligned} \tag{14}$$

with condition that,

$$f_2>\alpha_E. \tag{15}$$

(d) Equilibrium point $\mathcal{U}_4\left(C_4,E_4,L_4,A_4,0\right)$, such that

Table 1
Numerical values of the parameters.

Parameters	Baselines	Ranges	Source	Estimates
λ_L	0.1500		[27]	
λ_E	0.3700		[27]	
α_E^{-1}	7 $days$	[2 – 9] $days$	[38]	
δ^{-1}	14 $days$	[8 – 16] $days$	[8,38]	16 $days$
λ_A^{-1}	18 $days$	[15 – 18] $days$	[8,38]	
K_C	30 leaves $Plant^{-1}$		[27,40]	
K_E	10^4	$[10^5 - 10^8]$	–	10^6
η	0.03 day^{-1}	[0.01 – 0.07] day^{-1}	–	0.02 day^{-1}
ψ	0.04 day^{-1}	[0.02 – 0.7] day^{-1}	–	0.07 day^{-1}
ω	6×10^{-5} day^{-1}	$[6\times10^{-8} - 6\times10^{-4}]$ day^{-1}	–	6×10^{-6} day^{-1}
q	160		[38]	
r	0.05		[27]	
β_h	0.03	[0.02 – 0.07]	–	0.04

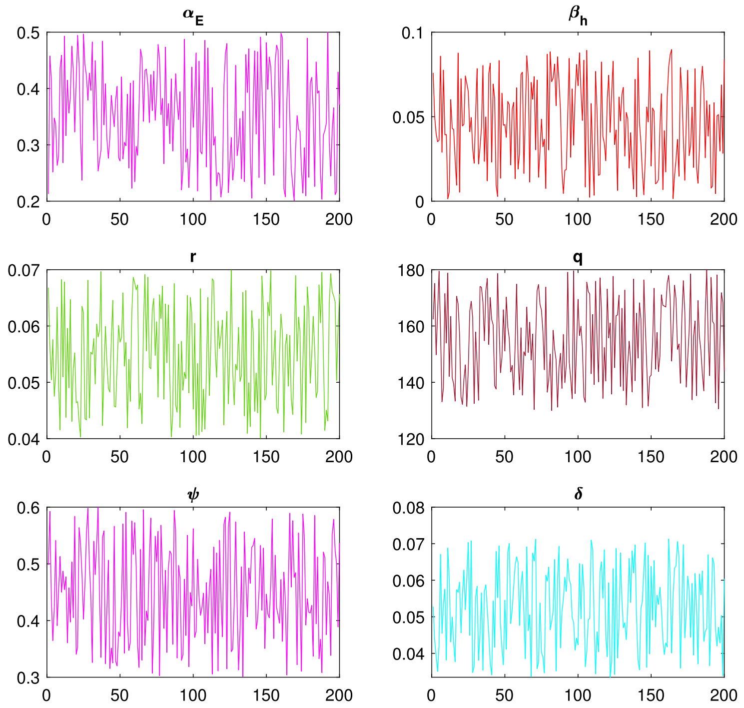

Fig. 3. Describes that the parameters ψ, δ, r, β_h and α_E are within the range.

$$\begin{cases} C_4 = \frac{f_2-\alpha_E}{\eta\omega}, \\ E_4 = \dfrac{K_E\psi qr\delta\left(K_C\eta\omega + f_2 - \alpha_E\right)}{K_E K_C\eta\omega^2 f_1\lambda_A + \psi qr\delta\left(K_C\eta\omega + f_2 - \alpha_E\right)}, \\ L_4 = \frac{r\left(K_C\eta\omega+f_2-\alpha_E\right)}{\eta\omega^2 K_C}, \\ A_4 = \frac{r\delta\left(K_C\eta\omega+f_2-\alpha_E\right)}{\eta\omega^2 K_C\lambda_A}. \end{cases}$$

(e) Equilibrium point $\mathscr{U}_5\left(C_5, E_5, L_5, A_5, H_5\right)$, where

$$\begin{cases} C_5 = \left(\frac{\omega\sigma_h}{r\beta_h\mu_h} + 1\right)K_C, \\ E_5 = \frac{\psi q\sigma_h\delta K_E}{\mu_h\beta_h f_1\lambda_A K_E+\psi q\delta\sigma_h}, \\ L_5 = \frac{\sigma_h}{\beta_h\mu_h}, \\ A_5 = \frac{\delta\sigma_h}{\beta_h\mu_h\lambda_A}, \\ H_5 = \frac{r\mu_h\left(K_C\eta\omega\beta_h+f_2\right)-\left(r\mu_h\alpha_E+\sigma_h\eta\omega^2K_C\right)}{r\mu_h{\beta_h}^2}, \end{cases}$$

provided that,

$$r\mu_h\left(K_C\eta\omega\beta_h + f_2\right) > \left(r\mu_h\alpha_E + \sigma_h\eta\omega^2K_C\right). \tag{16}$$

Therefore, the equilibrium points $\mathscr{U}_1$, $\mathscr{U}_2$, $\mathscr{U}_3$, and $\mathscr{U}_4$ remains positive if the condition stated in Eq. (15) holds. Also, the equilibrium point $\mathscr{U}_5$ exists if the condition stated in Eq. (16) remain true.

3. Parameter estimation of the basic model

In this section, we focus on parameter estimation, a crucial and critical aspect in improving the accuracy of quantitative predictions for time-based problems with real-world data. Specifically, we address the issue of *Plutella xylostella* infestations in cabbage production, aiming to estimate numerical values that will assist us to understand its dynamics and effectively manage its infestations in a cabbage farm. In the process of estimation, we utilized the Least Square Algorithm (LSA). In comparison to Maximum Likelihood Estimation, the LSA stands out for its ease of application and robustness, making it a straightforward and resilient choice.

To implement the Least Squares Algorithm for our model system (6), we utilize numerical values obtained from various literature sources as initial guesses to generate a noise data sets that reflects real data. To achieve this, Gaussian noise with a mean ($\mu = 0$) and standard deviation

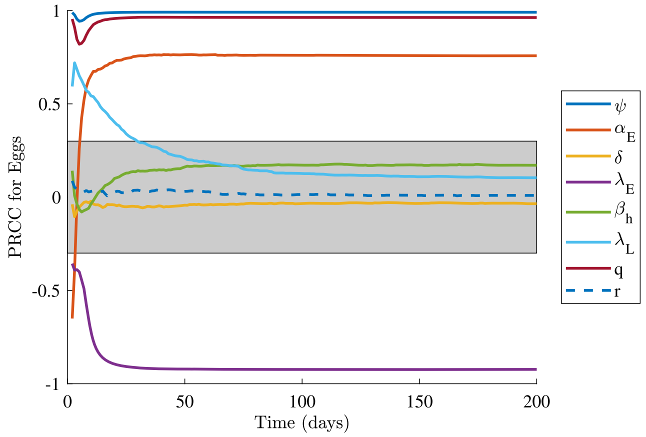

Fig. 4. Illustrations of the global variations of the model parameters with respect to Egg stage.

($\sigma = 1$) was added to the literature data points. We initiated the process by calculating the solution of literature data (L_i) using the Eq. (17). The model solution with noise data is represented by G_i with its solution captured by the Eq. (18), where $i = 1, 2, 3, \ldots n$.

$$L_i = f\left(x_i, \theta\right) \tag{17}$$

$$G_i = L_i + \text{Gaussian Noise} \tag{18}$$

To get the optimal parameter values with minimum residuals, we calculated the sum of squared residuals, which is obtained by taking the difference between observed and expected values.

The Eq. (19) describes the sum of squared residuals of our model system (Eq. (6)), where θ is the set of the parameters of our choice;

$$\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{n} \left(G_i - f\left(x_i, \theta\right) \right)^2. \tag{19}$$

After we are done with the estimation process, we have summarized our model parameters shown in Table 1.

4. Numerical visualizations

4.1. Global sensitivity of the model parameters

In this part, we investigate how the model responds to variations in each parameter within a specified uncertainty range, using the baseline values provided in Table 1. We employ the Latin hypercube sampling method to derive the partial rank correlation coefficients (PRCC). Before checking the sensitivities of the parameters of interests, we checked if they fall within the given range, as clearly demonstrated in Fig. 3.

The graphical results in Fig. 4 describes the robust positive influence of α_E and δ with the life cycle stage of *Plutella xylostella*. On the other hand, the scatter plots shown in Fig. 5 exemplifies the strong positive correlation of parameters α_E, ψ and q with the growth of eggs. The Fig. 7 clearly shows that, the parameters α_E and δ has strong positive correlation with the growth of larvae and the adult moths respectively. Conversely, the Fig. 6 shows that the attack rate (β_h) has a strong negative influence to the development of larva and adult moth. If we vary the value of β_h we will minimize the population of moth infestation in the cabbage biomass.

In this regard, to reduce the outbreak of moths stages in the cabbage biomass, we have to apply the intervention strategies to the more sensitive parameters or with positive influence to the growth of larva and adult moths. The interventions such as pest control campaign, habitat management and applicable IPM methods for *Plutella xylostella* control.

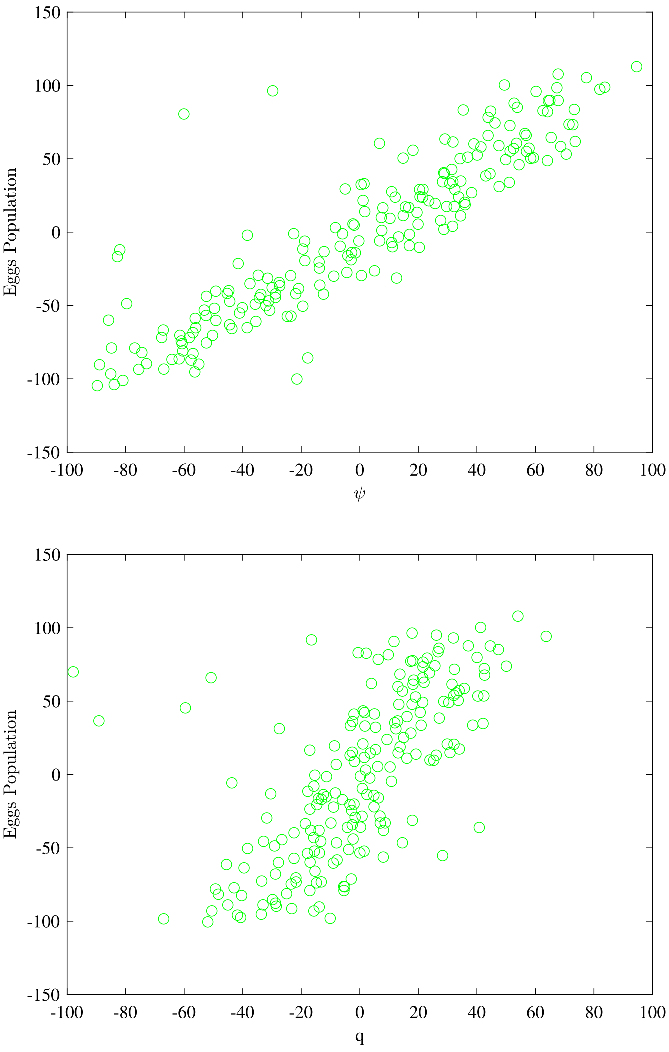

Fig. 5. The parameters α_E, q, and ψ has strong positive correlation to the growth of egg population.

5. Discussions of the results

This section discusses the estimation of parameters for a proposed mathematical model involving predators, cabbage biomass, and the life cycle of *Plutella xylostella*. The estimation process begins with literature values as initial guesses. Later, Gaussian noise is introduced to these literature values, and subsequently, the residuals are minimized to achieve a well-fitting model. We have generated figures by using the initial conditions: $C(0) = 5$, $E(0) = 100$, $L(0) = 100$, $A(0) = 100$, $H(0) = 150$ to verify the entire process of estimating parameters and model fitting, as described below.

In Figs. 8(a)–(e), we present the best fits of the synthetic data generated using the estimated parameter values. Moreover, the autocorrelation of residuals for these estimated values falls within the range that signifies the observed correlation within the residuals.

To assess the validity of the estimated parameter values, we examine the nature of the distribution of their residuals across all model outputs. The results affirm their validity, as these residuals exhibit a normal distribution, by using the Histogram of residuals, Quantile-Quantile Plots and Auto-correlation

The Fig. 9 describes the histogram of residuals which provides justification that the proposed model fitting and the estimated parameter values stem from the same probability distribution function.

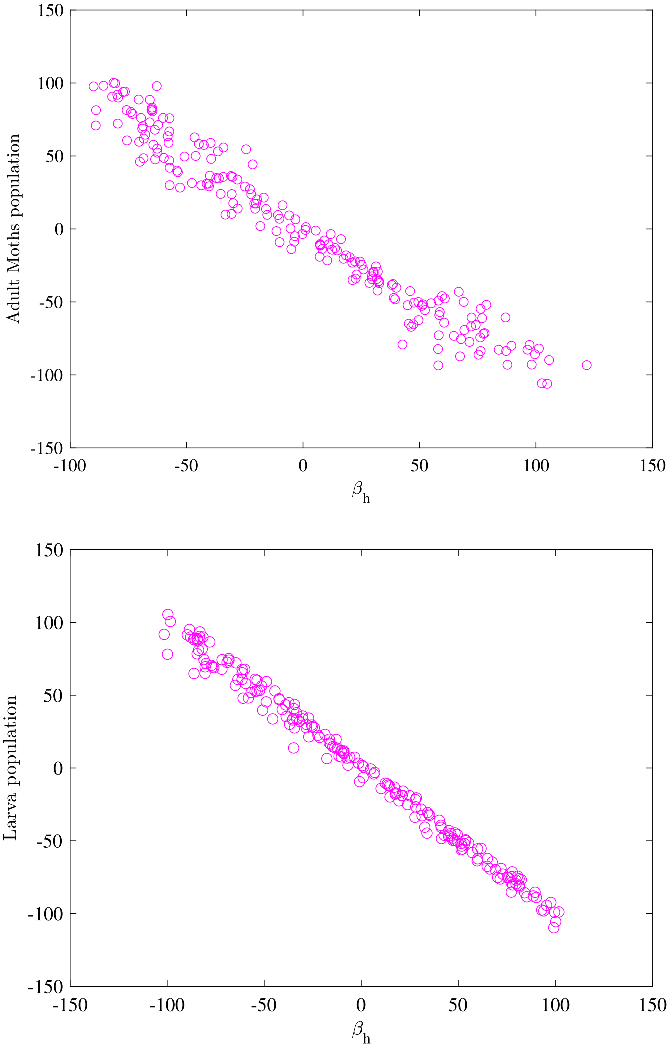

Fig. 6. Illustrates that predator attack rate (β_h) to larva negatively affects the growth of adult moth and Larva population.

A QQ plot, short for quantile-quantile plot, is a visual aid used for evaluating whether a given data sets could reasonably originate from a specific theoretical distribution, like the normal or exponential distribution [41]. It is constructed by graphing two sets of quantiles against each other. If both sets of quantiles are derived from the same distribution, we should observe the data points aligning closely to a straight line. Basing on our data set, it is well illustrated in Fig. 10 where both sets of quantiles genuinely follow a normal distribution confirming that our data comes from the same distribution. Also, Fig. 11 illustrates the auto-correlation values remain within an acceptable range for all 100 lags.

As both of these assumptions have been met, we successfully obtained the optimal numerical values for our parameters that minimize the squares of the residuals.

Therefore, basing on these estimations, we now utilize our estimated parameters to check the dynamics of diamondback moth infestations in cabbage biomass by performing the numerical visualizations. Finally, Fig. 12 (a)-(e), clearly demonstrates the ODE solution and the fitted data.

6. Conclusion

This paper presents a proposed nonlinear ODE model to study the dynamics and estimation of the parameters for diamondback moth infestations in a cabbage biomass using secondary data sources as the initial guess and adding Gaussian noise to the literature datasets based on the non-linear least-squares algorithm. This approach has been implemented to estimate parameters as it is ease and more robust than other method Maximum likelihood. A good agreement has been obtained between the simulation output and the literature data values, as well as the Gaussian noise data. The estimated parameters assist us in studying the dynamics of *Plutella xylostella* infestations in cabbage production. We also performed a global sensitivity analysis of all the parameters of interest to better understand which parameters influence the growth rates and development of *Plutella xylostella*, thereby hindering cabbage production. The parameters that are sensitive to the growth of *Plutella xylostella* are ψ, α_E and δ. Therefore, this study recommends that if we want to minimize the population of *Plutella xylostella* to an acceptable level, controls should be applied to these parameters. Hence, we maximize cabbage production.

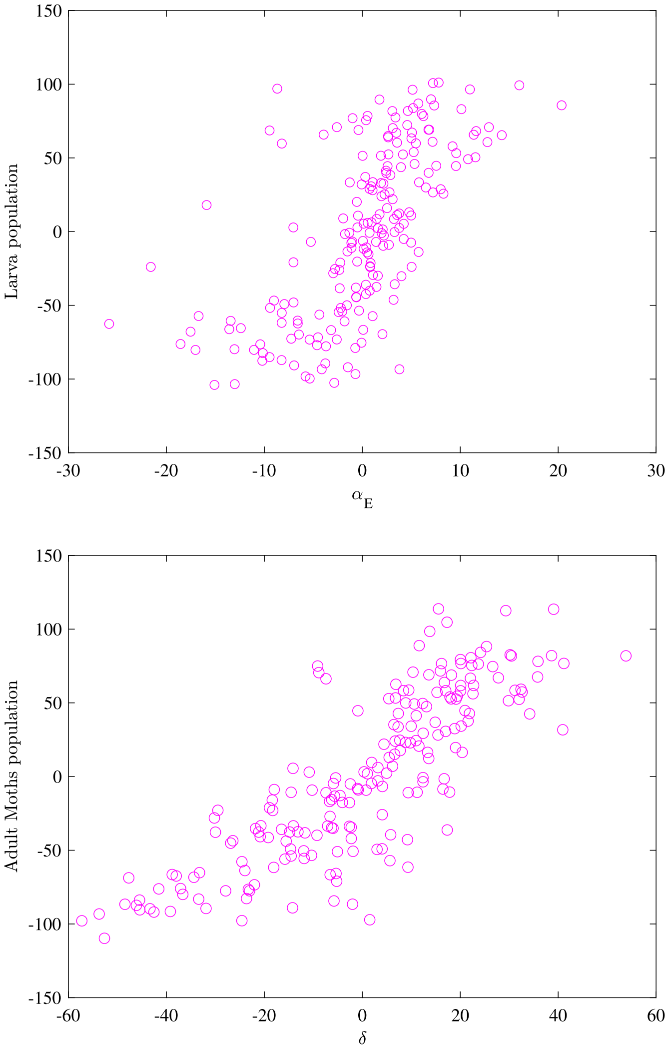

Fig. 7. Describes the parameters α_E and δ has positive influence to Larva and Adult Moth population respectively.

This study is constrained by the unavailability of real data that could have been used for parameter estimation. Consequently, we opted to utilize White Gaussian noise data, which mirrors real-world conditions, to simulate the dynamics of the pest in a cabbage biomass.

Furthermore, this method provides the foundation for estimating parameters to our proposed model system. In future research, we aim to explore fractional-order differential equations as studied by Nosrati Firoozsalari et al. [42], Aghaei and Parand [43]. On top of that, as we are in the era of Artificial intelligence and the prevalence application of machine learning in pest management, we plan to incorporate

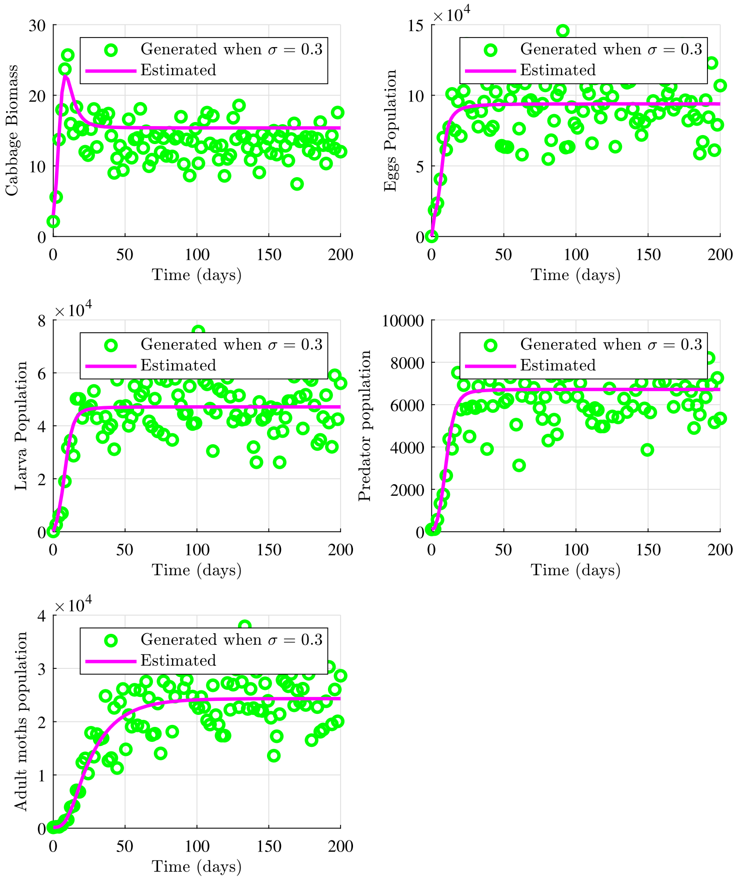

Fig. 8. Shows the noise data versus fitted solution.

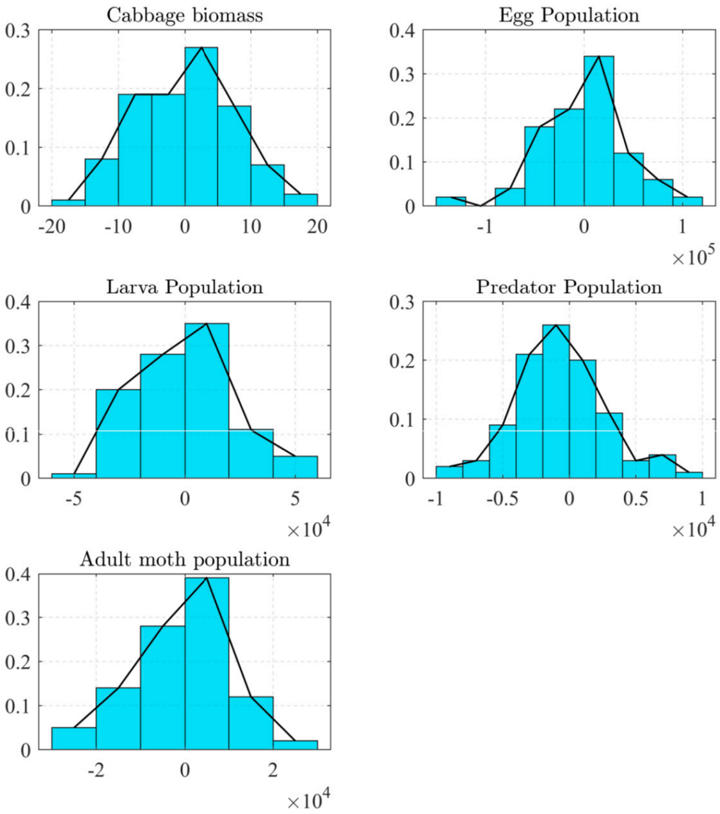

Fig. 9. Exemplifies the Histogram of residuals for standardized model parameters.

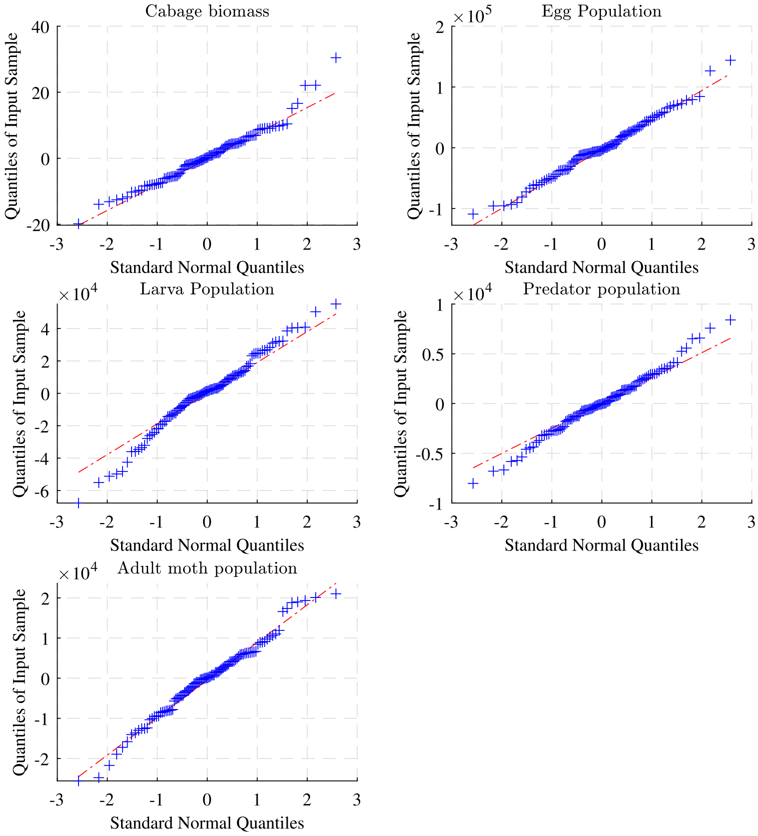

Fig. 10. The QQ-plots to describe the normalization of the model data.

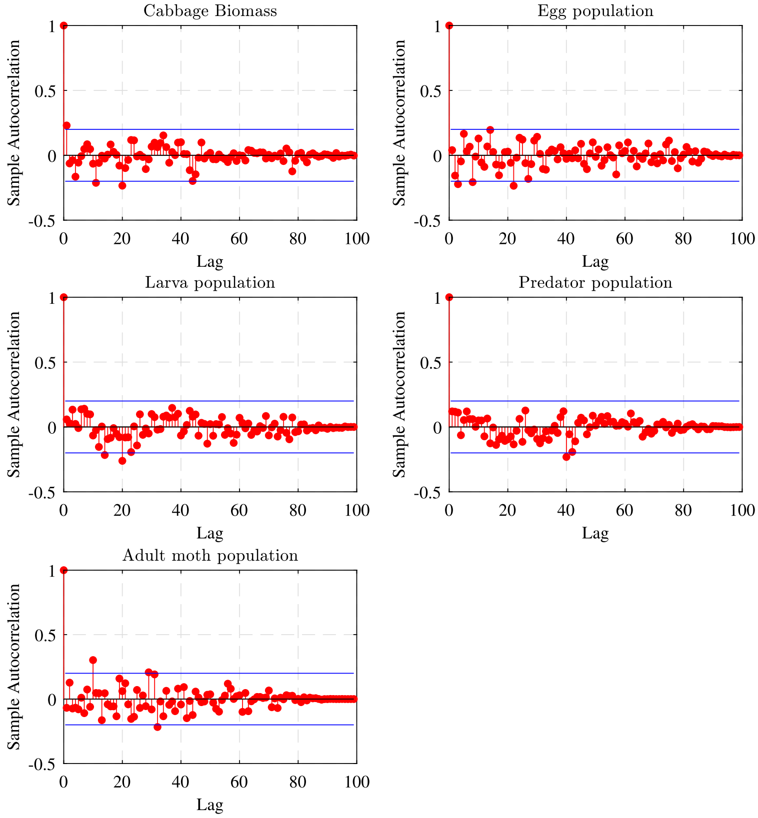

Fig. 11. The Auto-Correlation diagram describing the model validity of the model data.

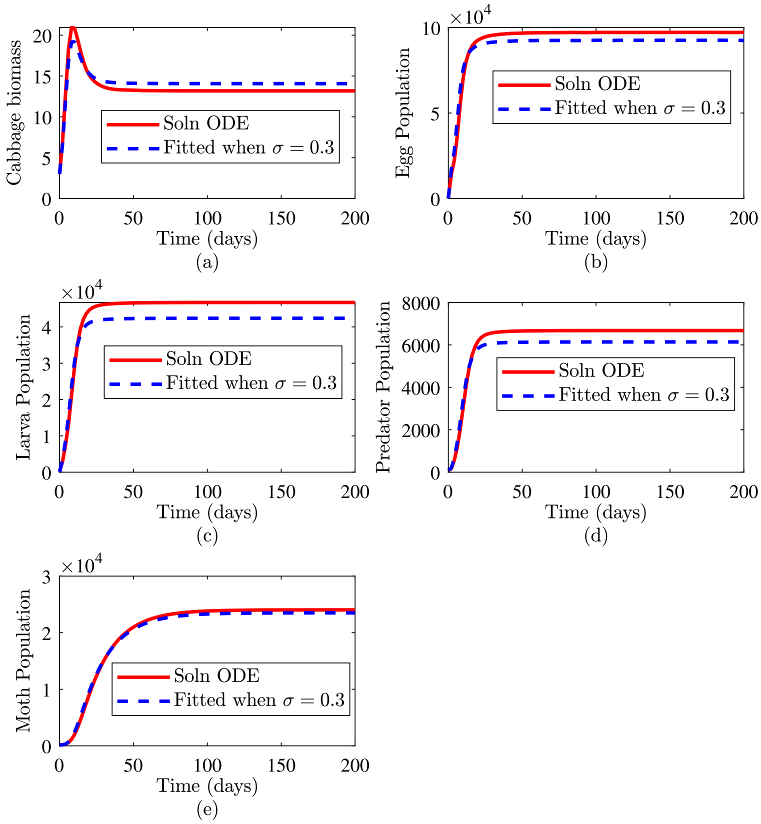

Fig. 12. Illustrates the ODE solution versus the fitted solution.

image segmentation algorithms that will assist us in knowing the level of infestations of *Plutella xylostella* in a cabbage biomass.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

CRediT authorship contribution statement

Daniel Paul: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Visualization, Writing – original draft. **Maranya Makuru Mayengo:** Supervision, Writing – review & editing. **Salamida Daudi:** Supervision.

Declaration of competing interest

The authors declare no personal connections or conflicts of interest.

Data availability

The data used is included in the manuscript.

Acknowledgements

We would like to convey our deep appreciation to all those who provided their inputs during the preparation of this work.

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